SIMPLE VIBRATION TEST

1. Fundamentals of Mechanical Vibration

In this experiment, you will learn about the fundamentals of the mechanical vibrations. The mechanical vibration can be defined as the *motion of the bodies which is displaced from its equilibrium position*. If the system is in equilibrium position and if you apply a force by changing its position, the system tries to come its equilibrium position by repetitive motions by translation or by rotation. This behaviour of the bodies is defined as mechanical vibration.

In general, the vibration systems consist of **three**elements :

1- Mass of the system (***m***)
2- Spring constant of system(***k***)
3- Damping coefficient of the system(***C***)

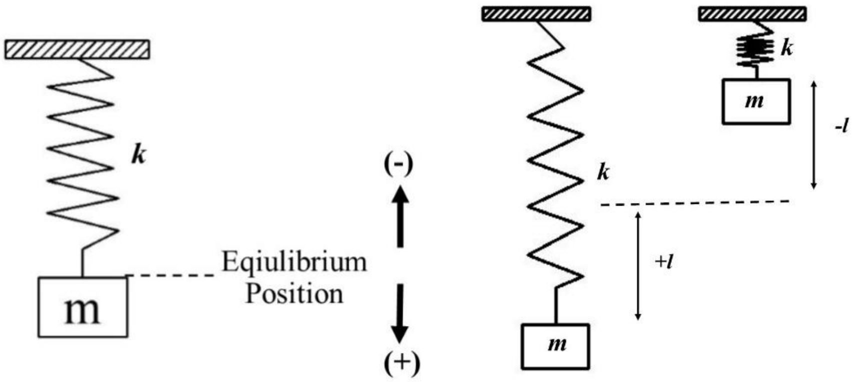


Figure 1. Simple spring mass system

In Figure 1, there is mass(***m***) and spring whose spring constant is ***k***, are hanged. The system is in equilibrium position. The movement direction is taken as positive(+) in downward direction and negative in upward direction(-). When you pull the mass downward and release it, the mass goes repetitively upward and downward. At the end of the movements (when its energy is consumed as kinetic energy), it stops its original(equilibrium) position.

The graphics of this mechanical movements is shown in the figure 1. The graphics shows the mass, "***m***", oscillates between the upper and lower amplitude"***l***". This type of motion is called harmonic motion. The harmonic motion (periodic motion or oscillation) can be described as the restoring force in the system is against the displacement taken during motion.

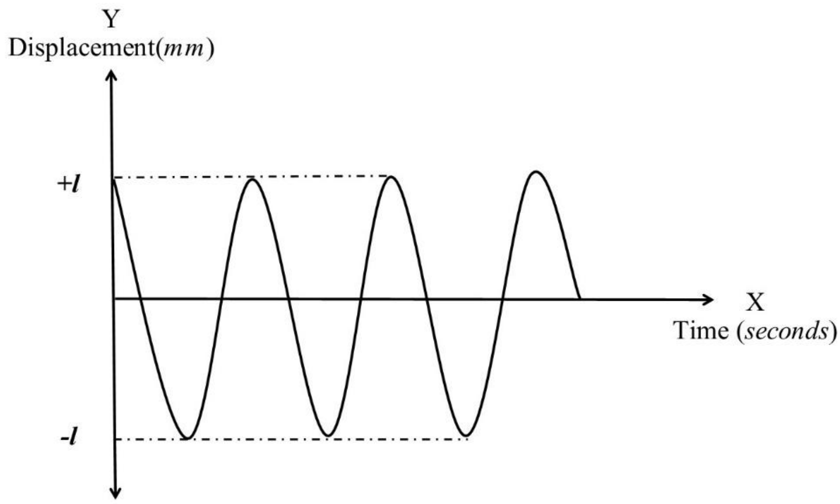


Figure 2. Graphics of simple vibration (*ideal system and conditions*)

The mathematical model of spring-mass system :

$$\boxed{m\frac{d^2s}{dt^2}+C\frac{ds}{dt}+ks=F(t)} \quad \text{Eq.1}$$

In Equation 1 :

m= mass of the system(*kg*)

C=damping coefficient of the system(*kg/s*)

k=spring constant(*N/mm*)

s=the displacement (*mm*)

F(t)= time dependent Force (*N*)

2.The Definitions used in the Vibration Systems

Frequency (*f*) :The amount of cycles that a system makes in a unit time. It is demonstrated by ***f***and very important in vibration phenomena. It is dependent on the spring stiffness (*k*) and mass of the system(*m*). Its unit is *1/second* or *Hertz(Hz)*.

Natural frequency(W_n) : Natural frequency is the frequency at which the system vibrates free vibration conditions. Namely, there is no any external forced on the system. It is shown by W_n and its unit is *Hz*. In the forced vibration condition, when the frequency of the force is equal to the natural frequency of the system, the amplitude of the vibration equals goes to the infinity. This event is called as *Resonance*.

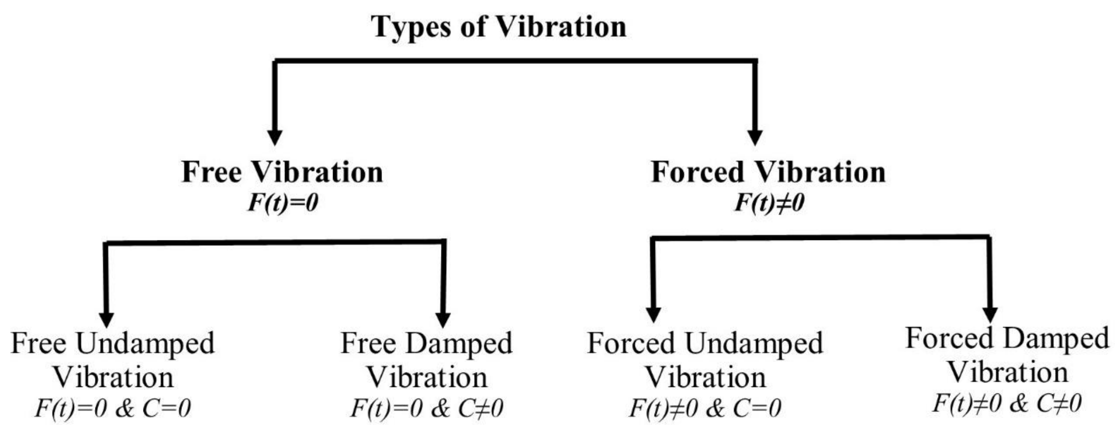
Period (*T*) :The period is the time passes for one cycle of the motion. It is demonstrated by ***T*** and its unit is seconds(*s*).

Damping Coefficient(C) and Damping Ratio(r) : This definitions are related to the damping. The damping means the resistance to the vibration. When the damping coefficient gets higher, the vibration amplitude and duration gets lower. The damping coefficient is demonstrated by ***C*** and its unit is *kg/s* , the damping ratio is shown by ***r*** and it is unitless.

3.Types of Vibration

The types of vibration mainly divided into two main categories as *Free Vibration* and *Forced Vibration*. The Free Vibration is the vibration which there is no any force, F(t), applied on the mass, namely **F(t)=0**. The Forced Vibration, in contrast, the vibration which a force, F(t) is applied on the mass. In this experimental setup we will see the Free Vibration only since we do not apply any F(t) during motion.

At the same time, these vibration types can be subdivided into two categories as *Undamped Vibration* and *Damped Vibration*. If there is a resisting force against vibration, the system is called Damped Vibration, if not the system is called Undamped Vibration.



3.1. Free Undamped Vibration (*F(t)=0 & C=0*)

This system only includes mass(*m*) and spring stiffness(*k*). There is <u>no damper</u> in the system. The mass is pulled and released. The motion of the system will be observed and the spring stiffness(*k*), frequency(*f*) and period(*T*) of the system will be calculated by both theoretical and experimental methods.

When F(t)=0 and C=0 , the equation becomes

$$\boxed{m * \frac{d^2 s}{dt^2} + k * s = 0} \qquad \text{Eq.2}$$

$\frac{d^2 s}{dt^2} + \frac{k}{m} * s = 0$ and the natural frequency $\boxed{W_n^2 = \frac{k}{m}}$

$$\boxed{W_n = \sqrt{\frac{k}{m}}} \quad \text{(Natural Frequency Equation)}$$

$$\boxed{s(t) = s_0 * \mathrm{Cos} W_n t} \text{(The Amplitude Function)}$$

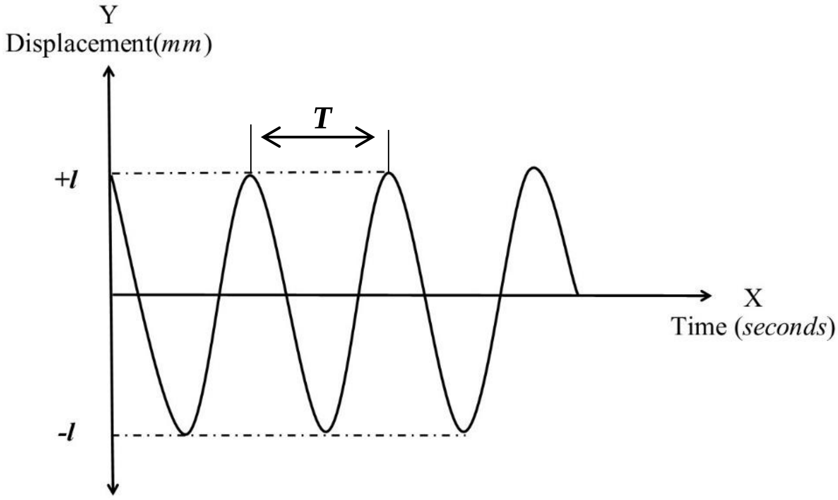


Figure 3. The Free Undamped Vibration Graph an period prediction (ideal condition)

$$T = \frac{1}{f} \text{ and } T = \frac{2\pi}{W_n} \ (sec) \ \leftrightarrow \ f = \frac{1}{T} \ (Hz)$$

3.2. Free Damped Vibration (*F(t)=0 & C≠0*)

In this system, a damper is included. A damper is a resistive environment for reducing vibration. In our experimental setup, a piston which has holes on it is moved in an oil cap. While the piston moves together with mass(*m*), the mass losses its energy in every cycle and the vibration is absorbed by this way. Namely, damping is produced by processes that dissipate the energy stored in the oscillation. The aim in this experiment is to obtain the damping coefficient(*C*) and damping ratio of the system(*r*).

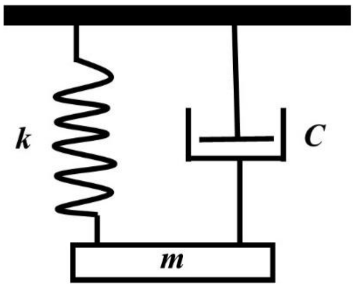


Figure 4. Basic Free damped system

The motion equation for this system is :

$$m*\frac{d^2s}{dt^2}+C*\frac{ds}{dt}+k*s=0$$

$$\frac{d^2s}{dt^2}+\frac{C}{m}*\frac{ds}{dt}+\frac{k}{m}*s=0$$

Or we can write he equation : $\boxed{\frac{d^2s}{dt^2}+2*W_n*\frac{ds}{dt}+W_n^2*s=0}$ Eq.3

where $\boxed{r=\frac{C}{2*W_n*m}}$

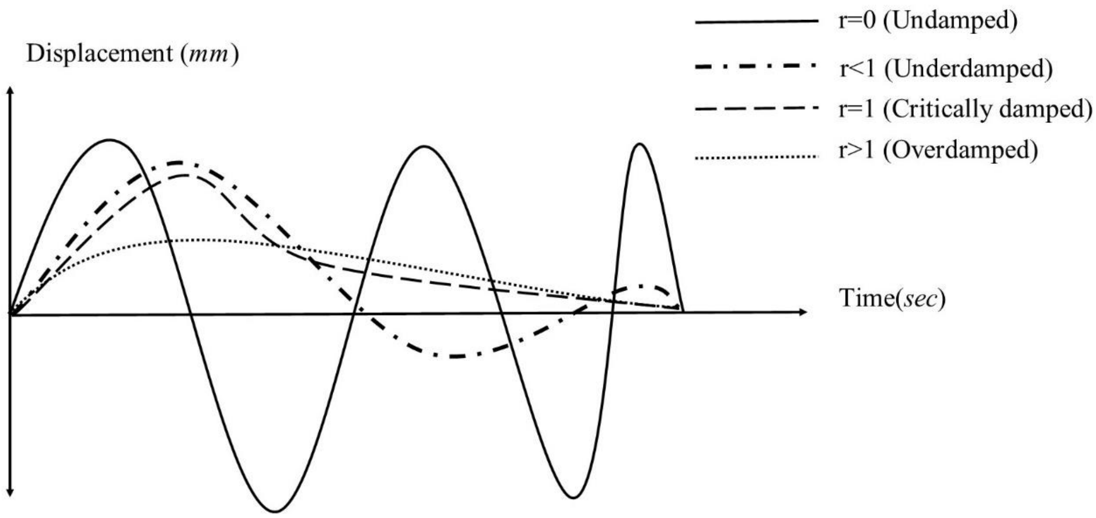


Figure 5. Free Damped Vibration Types

In Figure 5, The free damped vibration systems are shown. In the figure, actually there are four category. But first one is Undamped vibration showing that *r=0*. In free damped vibration, there are 3 situation as **Underdamped(*r<1*)** the roots are complex roots, **Critically damped(*r=1*)** the roots are equal and opposite sign and **Overdamped(*r>1*)** there are two distinct real roots. It depends on the solution of the differential equation of the motion. (Eq.3). The root of this equation defines the types of the equation.

a. If **r<1**It is called**Underdamped free vibration.** The equation of motion becomes

$$\boxed{s(t) = e^{-r*W_n*t} * (A * \mathrm{Cos} W_d * t + B * \mathrm{Sin} W_d * t)}$$

The damping ratio(r) can be calculated from: $\boxed{r = \frac{1}{2\pi} * \ln \frac{s_n}{s_{n+1}}}$

The damped period : $T_d = \frac{2\pi}{W_n * \sqrt{1 - r^2}}$

The damped natural frequency(W_d) : $\boxed{W_d = W_n * \sqrt{1 - r^2}}$

b. If **r=1** it is called **Critically damped free vibration**. The motion equation

$$\boxed{s(t) = (A + Bt) * e^{-r*W_n*t}}$$

c. If **r>1** it is called **Overdamped free vibration**. The motion equation becomes

$$\boxed{s(t) = A * e^{(-r+\sqrt{r^2-1})*W_n*t} + B * e^{(-r-\sqrt{r^2-1})*W_n*t}}$$

- *s(t)* is the displacement(amplitude) of the spring changing with time

Experimental Setup

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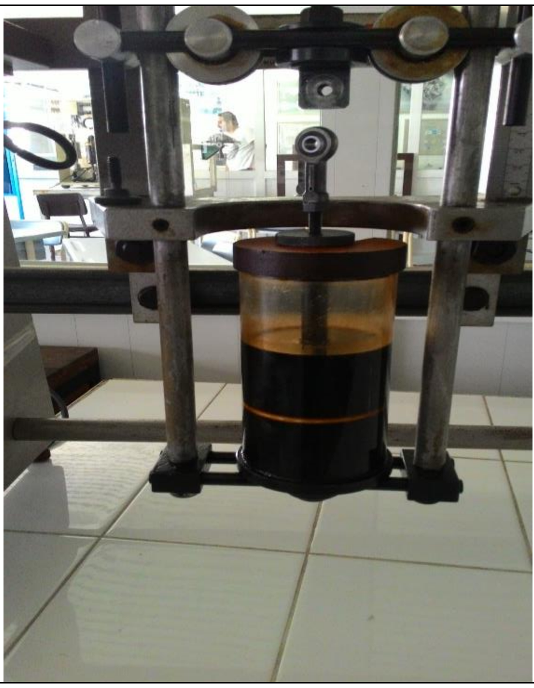

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Figure 6. Experimental Setup&Equipments
([1] *Spring*, [2] *Mass attached to spring*, [3]*Damper(oil pan)*, [4] *Whole experimental setup*).

The Graphs from Experiment

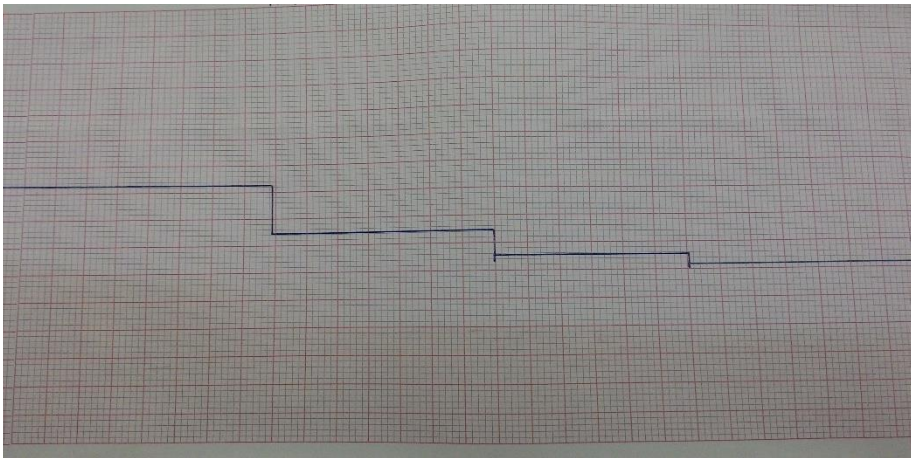

Figure 7. Calculation of spring stiffness, $\boldsymbol{k}$

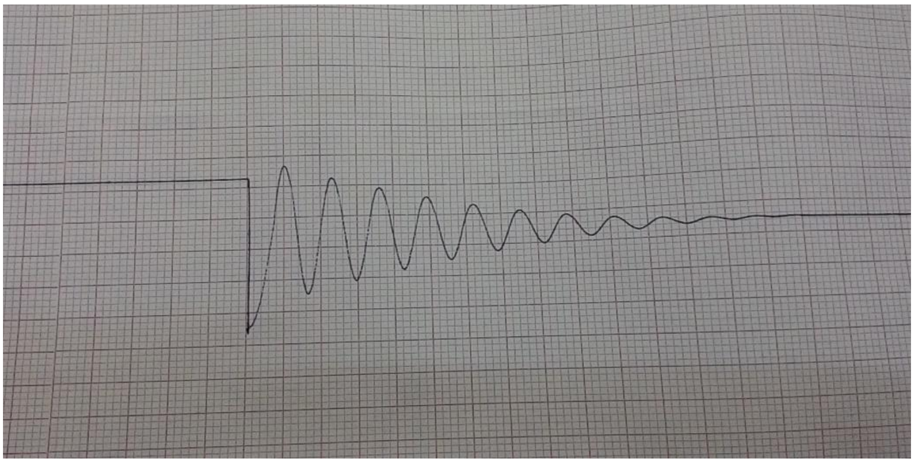

Figure 8. Calculation of period($\boldsymbol{T}$) and frequency($\boldsymbol{f}$)-Free Undamped Vibration

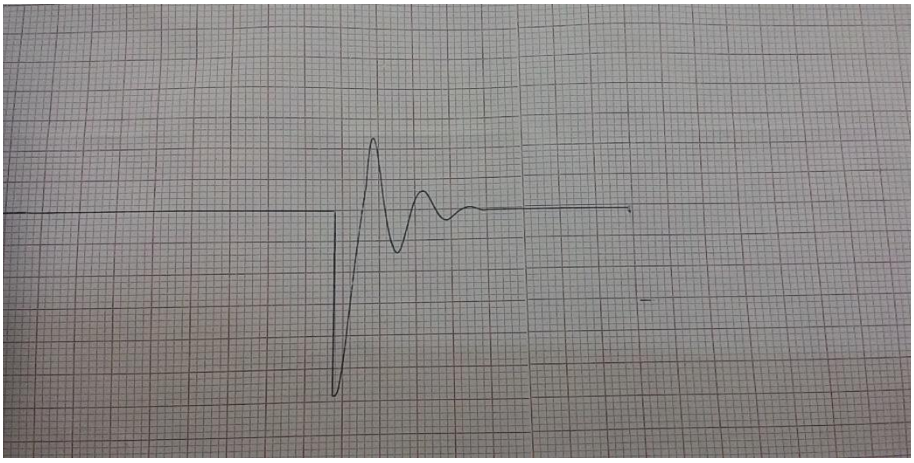

Figure 9. Calculation of damping ratio($\boldsymbol{r}$) -Underdamped Vibration

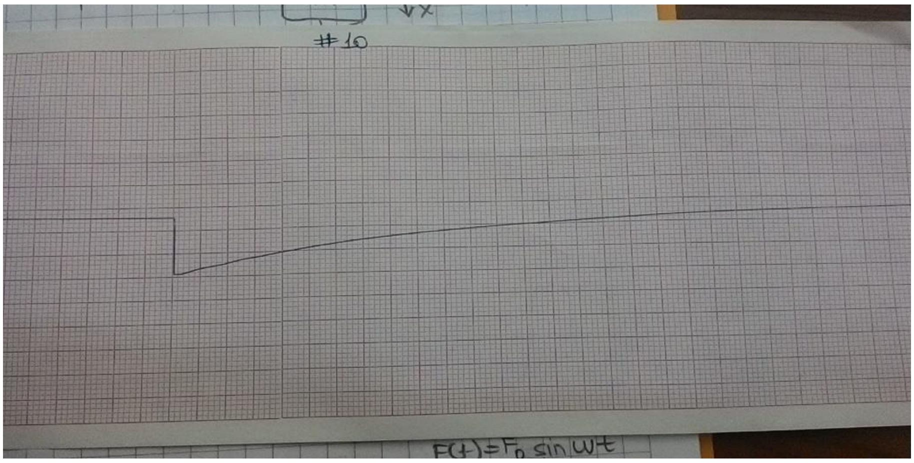

Figure 10. Calculation of damping coefficient(***C***)-Overdamped Vibration

Conclusions

In this experiment, the fundamentals of mechanical vibration are given. Some experiments are made and parameters of the vibration(***f***, ***T***, ***C***, ***r***) is calculated theoretically and experimentally. The student will learn more about other vibration types and improve himself/herself. The report requirements are as following questions :

Questions

1- Find the spring constant(k) of the system.
2- Determine the frequency(f) and period(T) of the undamped system under different masses.
3- Find the damping coefficient(C) and damping ratio(r) of the underdamped system.
4- Find the damping coefficient(C) of the overdamped system.
5- By using MS Excel draw the graphics according to *s(t)* function in free undamped system with changing the mass(m) and spring constant(k) with same amplitude.
6- Draw the graphics of mass(m)-natural frequency(W_n) & spring constant(k)-natural frequency(W_n) to show the effects of mass and spring constant on natural frequency.
7- Explain the *Resonance* event and give examples about it.